

MAXIMAL ACCELERATION OR MAXIMAL ACCELERATIONS?

Antonio Feoli*

Dipartimento d'Ingegneria, Università del Sannio,
Corso Garibaldi n. 107, Palazzo Bosco Lucarelli
82100 - Benevento - Italy

Abstract

We review the arguments supporting the existence of a maximal acceleration for a massive particle and show that different values of this upper limit can be predicted in different physical situations.

*E-mail: feoli@unisannio.it

1 Introduction

Since Weyl's attempt [1] to unify Gravitation and Electromagnetism, many different generalizations of Einstein Relativity have been proposed increasing the number of dimensions as in Kaluza - Klein approach or using Finsler spaces, complex manifolds, scalar - tensor coupling, extended particles in the form of strings or bubbles, etc. Some of these models start from fixing another observer independent scale in addition to the speed of light of standard Special Relativity. Recently a new interest in this kind of theories has been revived by Amelino-Camelia [2] who proposed a model based on the existence of a Length scale and by Magueijo and Smolin [3] who preferred to use an Energy scale. Both the attempts lead to a resulting maximal momentum (which has been found also by Low [4] using a different approach). Also Ahluwalia and Kirchbach [5] argued that the interplay of gravitational and quantum realms requires two invariant scales and obtained a gravitationally modified de Broglie wavelength that acquires a value of the order of Planck length in Planck regime. Another way was followed by Ketsaris [6] who, starting from a seven dimensional manifold, obtained a Maximal Acceleration and a Maximal Angular Velocity. The three models of "Quantum Special Relativity" need to be developed and Amelino-Camelia [7] suggests "to reach a formulation of a *Quantum General Relativity* by an appropriate extension of the k - Minkowski spacetime to some sort of k - phase space (which however here is intended as the space x_i, t, p_i, E rather than just x_i, p_i)". This was the same aim of the "Quantum Geometry" proposed by Caianiello [8] more than twenty years ago that led to the introduction of a maximal acceleration [9]. The boosts deformations of a "Quantum Special relativity" with an upper limit on the acceleration were derived by Scarpetta [10] in 1984 and even different models of "Quantum General Relativity" have already been developed by Brandt [11] and Schuller [12]. Caianiello himself and his co-workers have analyzed the quantum corrections to the classical spacetime metrics due to the existence of a Maximal Acceleration. His idea [13] is that the simplest

theoretical framework, which includes maximal proper acceleration, consists of considering as physical invariant, not the classical four dimensional space time distance element, but a new one, more general, defined in an eight dimensional phase space, where the infinitesimal element of distance can be written

$$d\tilde{s}^2 = g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu + \frac{c^4}{A_{max}^2}g_{\mu\nu}d\dot{x}^\mu d\dot{x}^\nu. \quad (1)$$

where $\dot{x}^\mu = dx^\mu/ds$ is the relativistic four-velocity. The consequence is that a particle of mass m accelerating along its worldline, behaves dynamically as if it were embedded in a spacetime of metric

$$d\tilde{s}^2 = ds^2 \left(1 - \frac{c^4 |g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu|}{A_m^2} \right), \quad (2)$$

But which is the right value of A_m to use in (2)? From the historical point of view the maximal proper acceleration has been first derived starting from the principles of Quantum Mechanics and Relativity by Caianiello [14] who obtained the value

$$A_{max} = \frac{2mc^3}{\hbar} \quad (3)$$

depending on the rest mass of the particle m . While in the context of quantum geometry [8] the maximal acceleration is generally referred to extended particles, the proof [14], from the Heisenberg uncertainty principle, holds also for point particles. Starting from the value (3), we obtained interesting results both with a simplified model (lacking of covariance) applied to Rindler [13], Schwarzschild [15], Reissner - Nordstrom [16], Kerr [17] and Robertson - Walker [18] metrics, and with a fully covariant approach that leads to a complete integrability of equations of motion (up to now) only in spacetimes of constant curvature [19]. As the concept of maximal acceleration has proved to be very fertile producing a lot of different interesting models, in this paper we want to review critically the main arguments that support the *existence* of this upper limit but not its *uniqueness*. We will quote old results and will give a new interpretation to some recent papers showing that, after Caianiello's proposal, different values of Maximal Acceleration can be predicted in different physical situations.

2 Maximal acceleration for extended objects

It is well known that massive extended objects imply critical accelerations, determined by the extension of the particles and by the causal structure of the space-time manifold. For instance, in classical relativity [20], an object of proper length λ , in which one extreme point is moving with acceleration a with respect to the other, will develop a Rindler horizon at a proper distance a^{-1} from the accelerated extremity, so that all parts of the object can be causally connected only if $\lambda < a^{-1}$. This implies a proper critical acceleration $a_c \simeq \lambda^{-1}$ which depends on λ and diverges in the limit in which the object reduces itself to a point-like particle.

In the quantum relativistic context, the analysis of string propagation in cosmological backgrounds revealed that an acceleration higher than the critical one give rise to the onset of Jeans-like instabilities [21] in which the string oscillating modes develop imaginary frequencies and the string's proper length diverges. Gasperini [22] has given a very interesting kinematic interpretation of this string instability, showing that it occurs when the acceleration induced by the background gravitational field is large enough to render the two string extremities causally disconnected, because of the Rindler horizon associated with their relative acceleration. This critical acceleration a_c is determined by the string size λ and is given by $a_c = \lambda^{-1} = (m\alpha')^{-1}$ where m is the string mass and α'^{-1} the usual string tension.

Frolov and Sanchez [23] analyzed the dynamics of an uniformly accelerated open string in flat space. They used the classical Rindler metric

$$ds^2 = -\xi^2 d\eta^2 + d\xi^2 + dy^2 + dz^2 \quad (4)$$

(where the Rindler coordinates are $\xi = 1/a$, $\eta = as$, and a is the acceleration) and supposed that there are two heavy particles (e.g. monopoles) at the ends of the string, numbered with the indices 1 and 2, on which some external force is applied in such a way that both particles are moving with the same constant proper acceleration $a = g$. In an inertial frame of reference the coordinates (t, x, y, z) are chosen in such a way that the x -axis coincides

with the direction of acceleration, while the y -axis is parallel to the distance L between the ends of the string; correspondingly, the Rindler coordinates are (η, ξ, y, z) and in the accelerated Rindler frame the particles at the string ends obey the boundary conditions:

$$\xi_1 = \xi_2 = g^{-1}; \quad y_1 = -y_2 = L/2; \quad z_1 = z_2 = 0 \quad (5)$$

Putting $y = L\sigma/\pi$, the spatial parameter σ varies from $-\pi/2$ to $\pi/2$.

They found a special solution of the equations of motion describing an uniformly accelerated string, which moves as a rigid body without any excitation:

$$\xi = \frac{L}{\pi\beta} \cosh\left(\frac{\beta\pi}{L}y\right) \quad (6)$$

As for the β parameter, its value is fixed by the boundary condition that the string ends must move with the assigned acceleration g , expressed by the equation:

$$\cosh\left(\frac{\beta\pi}{2}\right) = \frac{\beta\pi}{gL} \quad (7)$$

For different values of the acceleration g , this equation admits two, one, or no solution for β . Frolov and Sanchez proved that rigid equilibrium configurations of the accelerated string exist only for an acceleration less than a critical one. Finally, they calculated the string size λ in the Rindler frame:

$$\lambda = \frac{2L}{\beta\pi} \sinh\left(\frac{\beta\pi}{2}\right) = \frac{2L}{\beta\pi} \sqrt{\cosh^2\left(\frac{\beta\pi}{2}\right) - 1} \quad (8)$$

Now, in order to compare their result with Gasperini's one, we can substitute (7) in (8) and easily obtain the parameter β in terms of the acceleration and the proper length of the string:

$$\beta = \pm \frac{2gL}{\pi\sqrt{4 - g^2\lambda^2}} \quad (9)$$

From the equation above we calculate that β is real if

$$g < \frac{2}{\lambda} \quad (10)$$

that confirms the critical value of acceleration predicted by Gasperini.

Papini, Wood and Cai [24] showed that the same maximal acceleration (10) of an extended particle follows naturally from the theory of conformal transformations. On the other side, they also studied [25] the motion, in a sort of Madelung fluid, of a spherically symmetric extended object, a bubble, of Riemannian geometry embedded in external Weyl geometry where a conformal covariant calculus is used. The field equations for that case are obtained starting from the conformally invariant action

$$I_C = \int \left\{ -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} + |\beta|^2 \bar{R} + k | \bar{\square}_\mu \beta \bar{\square}^\mu \beta | + \lambda |\beta|^4 \right\} \sqrt{-g} d^4x \\ + \int \rho g^{\mu\nu} \gamma_\mu (\bar{\square}_\nu \rho - \epsilon \rho \varphi_{,\nu}) \sqrt{-g} d^4x, \quad (11)$$

where β is a complex scalar field with $\rho = |\beta|$ and $\varphi = \arg \beta$, then γ_μ is a vector Lagrange multiplier and k and λ are arbitrary constants. Furthermore an overbar is used to distinguish an object defined in terms of the gauge-covariant calculus of Weyl geometry from the corresponding object associated with the covariant calculus of Riemannian geometry. For example, $\bar{\square}$ is the spacetime gauge-covariant derivative. Even in this very different case they found a maximal acceleration for the bubble

$$A_{max} = \frac{2}{R} \quad (12)$$

(where R is the bubble's radius) very similar to the value (10).

In any case we can conclude that the maximal acceleration depends on the characteristic size of the extended object we are considering.

3 Maximal acceleration for classical charged particles

Recently Goto [26] has studied the equation of motion for a classical charged particle including radiation reaction force performing a stable hyperbolic motion immersed in a uniform gravitational field. He finds that the observer in the laboratory frame measures the gravitational force acting on charged

particles as

$$F_g = \frac{mg}{1 - g^2\tau^2} \quad (13)$$

where

$$\tau = \frac{2e^2}{3mc^4} \quad (14)$$

His interpretation of equation (13) is that the gravitational mass of charged particles should be slightly greater than its inertial mass. But we can write his formula also as

$$F_g = \frac{mg}{1 - g^2/A_{max}^2} \quad (15)$$

and interpret it considering that an infinite gravitational force F_g is necessary to produce an acceleration

$$g = A_{max} = \frac{3mc^4}{2e^2} \quad (16)$$

In this case the maximal acceleration depends on the charge of the classical particle.

It is worth noting that Caldirola [27], in his theory of the classical electron founded on the introduction of a fundamental interval of time (the so called chronon), showed the existence of a maximal value of acceleration equal to half of Goto's one (16).

4 Maximal acceleration for nonspreading wave packets

Recently Caldas and Silva [28] have analyzed the motion of a nonspreading wave packet in a harmonic potential. It is well known that in the case of a quantum harmonic oscillator the motion of the center of a wave packet is rigorously identical to that of a classical particle. Caldas and Silva impose that the packet does not spread so that

$$[\Delta q(t)]_{timeaverage}^2 = [\Delta q_o]^2 \quad (17)$$

from which they obtain [28]

$$\Delta p_o = m\omega\Delta q_o \quad (18)$$

and assume that, at the initial time, the wave packet is such that

$$\Delta q_o \Delta p_o = \frac{\hbar}{2} \quad (19)$$

This way

$$m\omega(\Delta q_o)^2 = \hbar/2 \quad (20)$$

But considering that the classical particle (hence the peak of wave packet) obeys $q(t) = A\cos\omega t$, we have

$$v_{max} = \omega A \leq c \quad (21)$$

and

$$a_{max} = \omega^2 A \quad (22)$$

From (20) (21) and (22) we obtain the relation:

$$a_{max} \leq \frac{\hbar c}{2m\ell^2} \quad (23)$$

where ℓ is the characteristic size of the packet Δq_o . *In this case we find a maximal acceleration that depends on the square length of the wave packet.*

On the contrary, Caldas and Silva calculate a "classical variance" $\Delta q = A^2/2$ and require that it can be identified with the quantum variance, obtaining from (20)

$$m\omega A^2 = \hbar \quad (24)$$

From (22) and (24) and putting $\omega A = c$ they find a maximal driving force that we can read as a maximal acceleration

$$A_{max} = \frac{mc^3}{\hbar} \quad (25)$$

similar to Caianiello's one (3).

5 Maximal acceleration from maximal temperature

It is very easy to derive the existence of a maximal acceleration from a maximal temperature by using Unruh and Davies demonstration [29] stating

that a particle-detector subject to a constant acceleration would react to vacuum fluctuations as if it were at rest within a gas of particles having a temperature proportional to acceleration

$$T = \frac{\hbar a}{2\pi k c} \quad (26)$$

where k is Boltzmann constant.

Brandt [30], for example, starts from the result obtained by Sakharov [31], according to which the absolute temperature of thermal radiation in vacuum is

$$T_{max} \simeq \frac{c^2}{k} \sqrt{\frac{\hbar c}{G}} \quad (27)$$

that in (26) implies that there is a maximal acceleration relative to vacuum:

$$A_{max} \simeq \sqrt{\frac{c^7}{\hbar G}} = \frac{m_p c^3}{\hbar} \quad (28)$$

It is worth noting that A_{max} is similar to maximal acceleration found by Caianiello, but in this case the rest mass is substituted by the Planck mass $m_p = (\hbar c/G)^{1/2}$. *In this case the maximal acceleration is a universal constant* and assumes an extraordinarily high value: $A_{max} \simeq 5 \times 10^{53} cm/s^2$. Therefore it is very difficult to find through experimental tests some physical effects which can be ascribed to the existence of this upper limit on the acceleration.

The same demonstration can be performed starting with other values of maximal temperature available in literature [32]. For example, another interesting critical value of the temperature is the so called Hagedorn temperature $T_H \propto \alpha'^{-1/2}$ that arises also in string thermodynamics. Parentani and Potting [33] studied the motion of a string in Rindler frame and found the occurrence of a maximal temperature $T_{max} = T_H/\pi$ above which the string partition function diverges. Substituting this value of maximal temperature in the Unruh formula (26), we can find the same maximal acceleration of Gasperini.

6 Conclusions

The maximal acceleration principle can be successfully used to prevent the occurrence of singularities in General Relativity [16 - 19], and of ultraviolet divergences in quantum field theory [34], in particular in the estimation of free energy and entropy of quantum fields [35]. We have shown that several possible values of maximal acceleration can be found. The choice among them is crucial to obtain the right model of a relativistic dynamics with an upper limit on the acceleration and can be definitely done only through experiments. Finally it is even possible that two different values of maximal acceleration can survive in the same model. Using in (1) Planck acceleration (28), we obtained [36] a modified Rindler metric

$$ds^2 = -(\xi^2 - A_m^{-2})d\eta^2 + d\xi^2 + dy^2 + dz^2 \quad (29)$$

We showed that, in the case analyzed by Frolov and Sanchez, a maximal acceleration depending on the string's length λ still exists and it does not diverge in the limit $\lambda \rightarrow 0$, but we have $a \rightarrow A_m = m_p c^3 / \hbar$. *As in classical relativity only particles with zero mass can move at the maximal velocity c , so in our theory only point particles can move at maximal acceleration A_m .*

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